**Barron’s Let’s Review Regents – Algebra I**

# Chapter 8: Exponential Equations

## 8.1 Solving Exponential Equations

An exponential equation is one where the variable is an exponent. An example of a one-variable exponential equation is . Examples of two variable exponential equations are and   
.

**Finding Solutions to Exponential Equations**

In a two-variable equation like , substitute values for x, and find the corresponding y to get the solution. For example, if , then .

**Math Facts**

When a number other than 0 is raised to the 0 power, it becomes 1, not 0.

**Solving for the Exponent in an Exponential Equation**

In a one-variable exponential equation where the exponent is unknown, isolate the exponential expression and then use guess and check.

**Example**

Solve for the equation .  
The last step was done by guess and check.

### Check Your Understanding of Section 8.1

1. Multiple-Choice
2. If and , solve for y.  
   **(2) 9**
3. If and , solve for y.  
   **(3) 54**
4. If and , solve for y.  
   **(3) 1**
5. If and , solve for y.  
   **(4) 5**
6. If , and , solve for y.  
   **(3)**
7. If and , solve for x.  
   **(4) 3**
8. If and , solve for x.  
   **(1) 7**
9. If and , solve for x.  
   **(2) -3**
10. If and , solve for y.  
    **(4) 8**
11. If and , solve for y rounded to the nearest hundredth.  
    **(1) 2.70**
12. Show how you arrived at your answers.
13. Phoebe put $500 into the bank. The amount of money she has after t years is determined by the equation . After 4 years, how much money will Phoebe have in the bank.  
      
    Using Scientific Calculator:
14. The population of a town after t years can be approximated by the equation   
    . (a) According to the formula, what will the population of the town be after 10 years? (b) In what year will the population become 14,065?  
      
    (a)   
      
    (b) 15th year
15. Zoe drinks a cup of coffee that has 100 mg of caffeine. The amount of caffeine in the bloodstream after t hours can be determined by the equation . How much caffeine will be left in her bloodstream after 20 hours?
16. Food that is 110 degrees is put into a 30-degree freezer. The temperature of the food is related to the number of hours the food is in the freezer by the equation   
    . Between which two hours will the food be 32 degrees?  
      
    **Between 10 and 11 hours.**
17. Daphne says that is always greater than . Julia says that this is not true and that sometimes is greater than . Which student is correct? Explain.  
      
    Julia is correct.  
      
    For , will be greater than .

x 5^x 6^x  
 -1 0.20000 0.16667

-2 0.04000 0.02778

-3 0.00800 0.00463

-4 0.00160 0.00077

-5 0.00032 0.00013

-6 0.00006 0.00002

## 8.2 Graphing Solution Sets to Two-Variable Exponential Equations

The solution set to a two-variable exponential equation, like , can be produced with a table or with a graphing calculator. The shape of the graph is not a line or a parabola but is a distinctive shape that looks a bit like a playground slide.

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A graph with a line drawn on it

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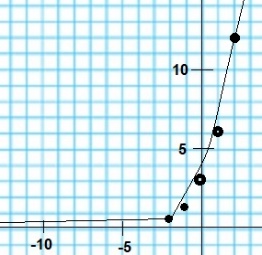
**Graphing the Solution Set to an Exponential Equation with a Table**

Five points are generally sufficient for graphing the solution set for an exponential equation. x-values of 2, 1, 0, -1, and -2 should produce enough points for an accurate graph.

**Example 1**

Make a table of solutions to , and use them to make a sketch of the graph of the solution set.

|  |  |
| --- | --- |
| x | y |
| 2 | 12 |
| 1 | 6 |
| 0 | 3 |
| -1 | 1.5 |
| -2 | 0.75 |



**Making Tables and Graphs on the Graphing Calculator**

**Example 2**

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AI-generated content may be incorrect.

A graph with a line

AI-generated content may be incorrect.

**Math Facts**

In an exponential equation, the thing being raised to the power is called the *base*. When the base of an exponential equation is between 0 and 1, the graph shows exponential *decay*. When the base is greater than 1, the graph shows exponential *growth*.

### Check Your Understanding of Section 8.2

1. Multiple-Choice
2. Which ordered pair is in the solution set of ?  
   **(4) (4, 81)**
3. Which ordered pair is in the solution set of ?  
   **(3) (3, 40)**
4. What is the graph of   
   Points: (-1, 0.5), (0, 1), (2, 4), (3, 8)**(3)**
5. Which is the graph of ?  
   Points: (-1, 2), (0, 1), (1, 0.5), (2, 0.25)  
   **(1)**
6. Below is the graph of which equation?  
   Points (-1, 0.33), (0, 1), (1, 3), (2, 9)  
   **(4)**
7. Below is a graph of  
   Points: (-2, 9), (-1, 3), (0, 1), (1, 0.33)  
   **(1)**
8. The chart below has ordered pairs for which equation?

|  |  |
| --- | --- |
| x | y |
| 0 | 6 |
| 1 | 12 |
| 2 | 24 |
| 3 | 48 |

**(4)**

1. The chart below has ordered pairs for which equation?

|  |  |
| --- | --- |
| x | y |
| 0 | 6 |
| 1 | 3 |
| 2 | 1.5 |
| 3 | 0.75 |

**(2)**

1. In what interval is the graph of increasing?  
   **(1) Always**
2. Below is the graph of . What is true about the value of b?  
   **(2) b must be less than 1 and greater than 0.**
3. Show how you arrived at your answers.
4. The graph of passes through the point (5, 243). What must the value of b be?  
     
   **Answer: b = 3**

|  |  |
| --- | --- |
| x | y |
| 1 | 1 |
| 2 | 32 |
| 3 | 243 |

1. After putting $200 into the bank, the amount of money Xavier has after 5 years is . Make a graph showing how the money grows for 5 years.

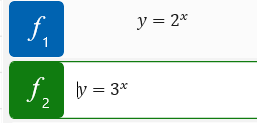
A graph with a line going up

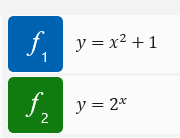
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1. Below is the graph of and on the same set of axes.  
     
   A graph of a function

   AI-generated content may be incorrect.  
     
   Is it true that for all values of x? Explain.  
     
   **No. for x less than 0.**

|  |  |  |
| --- | --- | --- |
| x | 2^x | 3^x |
| -1 | 0.50000 | 0.33333 |
| -2 | 0.25000 | 0.11111 |
| -3 | 0.12500 | 0.03704 |
| -4 | 0.06250 | 0.01235 |

1. What is the solution to this system of equations?  
   **Answer: x = 0, y = 1**  
     
   A graph of a function

   AI-generated content may be incorrect.
2. Below are the graphs of and   
   . They both contain the point (0, 1). Is it true that for all values of ? Explain.  
     
     
   A graph of a function

   AI-generated content may be incorrect.

The graphs intersect at points (0, 1) and (1, 2).  
From x = 0 to 1, .  
From x > 1, .  
  
Note: Book answer says x > 4.3, which appears to be incorrect.

## 8.3 Distinguishing Among Linear, Quadratic, and Exponential Equations

The graphs of the solution sets of linear, quadratic, and exponential equations are very distinctive. So it is possible to tell what type of equation a graph was A graph with a line and numbers in a chart

AI-generated content may be incorrect.produced from by just looking at it. When the solution set is in table form, it is possible to graph the data from the table or to determine which type of equation it came from with fast calculations.

**Identifying What Type of Equation It Is By Looking at the Graph of the Solution Set**

Linear equations have no exponents great than or equal to 1. The graph of a linear equation is always a line.

Quadratic equations have either an -term or a -term. The graph of a quadratic equation is a parabola.

Exponential equations have an x as an exponent. The graph of a exponential equation looks like a playground slide.

**Identifying That an Equation Is Exponential By Looking at a Table of Values**

**Using the Graphing Calculator to Graph Tables of Values**

**Example 4**

**Identifying the Type of Equation from a Table Without Graphing**

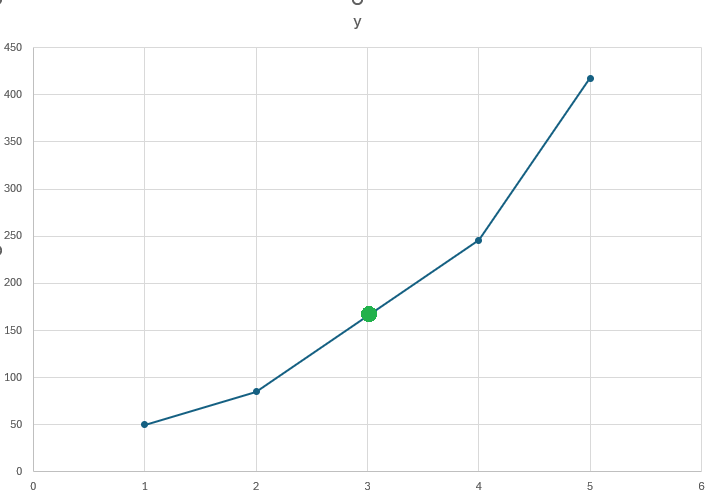
Since all the differences are the same, this is a linear equation.

A graph with a line and numbers in a chart

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### Check Your Understanding of Section 8.3

1. Multiple-Choice
2. What type of equation has a graph like the one below?  
   **(1) Linear**
3. Why type of equation has a graph like the one below?  
   **(1) Quadratic**
4. What type of equation has a graph like the one below?  
   **(2) Exponential**
5. Which of the graphs below corresponds to a quadratic equation?  
   **(3)**
6. Which of the graphs below corresponds to an exponential equation?  
   **(2)**
7. The ordered pairs in the following chart satisfy which type of equation?  
   **(1) Linear**
8. Which type of graph below corresponds to a linear equation?  
   **(1)**
9. The ordered pairs in the following chart satisfy which type of equation?  
   **(3) Exponential**
10. The ordered pairs in the following chart satisfy which type of equation?  
    **(4) None of the above**
11. The following scatterplot can be best modeled with which type of equation?  
    **(3) Quadratic**
12. Show how you arrived at your answers.
13. The population of a county for several different times is plotted on a graph where the x-axis represents years since the year 1990 and the y-axis represents the population in millions. The graph is below. What type of equation could be used to model this data? Explain.  
      
    An exponential equation can be best used to explain population increases, because population tends to increase by the same percentage each year.  
    The graph is increasing each year.
14. Some data points on a graph are plotted. Myah believes that this is a portion of a quadratic graph. Chloe believes that this is a portion of an exponential graph. Is there enough information to determine who is correct? If not, what more information would be needed?  
      
    Both Myah and Chloe could be correct, as the points could reflect that of a quadratic or exponential graph. There is not enough information.  
      
    More data about y-values for negative x-values would be helpful in determining whether it is a quadratic or exponential graph.
15. When the number of seats o na train is graphed as the y-coordinate and the time since 8:00 am is graphed as the x-coordinate, it makes the graph below. Which type of equation could be used to model this data? What details do you know about the equation besides just the type of equation it is?  
      
    It appears to be an exponential decay graph, because it is a curve that starts at a high value and decreases rapidly, approaching but never reaching the x-axis (horizontal asymptote), showing a decreasing trend over time.  
      
    For an exponential decay graph, the base will be between 0 and 1.  
      
    The starting value at time zero appears to be 100. It approaches nearly zero at 20 minutes.
16. Below is a chart with ordered pairs that satisfy an equation. The equation is linear, quadratic or exponential. Determine which type of equation it is, and fill in the missing number.  
      
    A screenshot of a calculator

    AI-generated content may be incorrect.  
      
    The type of equation is exponential.  
      
    The missing point is estimated to be (3, 150) to (3, 165).  
      
    
17. A tennis ball is dropped from a height of 30 feet. After each bounce the highest point is 80% as heigh as the bounce before. If the height of each bounce is graphed as the y-coordinate and the bounce number is the x-coordinate, will the graph be linear, quadratic or exponential.  
      
    The graph will be an exponential decay graph with the initial point of (0, 30).  
      
    A graph with a blue line

    AI-generated content may be incorrect.