**Barron’s Let’s Review Regents – Algebra I**

# Chapter 8: Exponential Equations

## 8.1 Solving Exponential Equations

An exponential equation is one where the variable is an exponent. An example of a one-variable exponential equation is . Examples of two variable exponential equations are and   
.

**Finding Solutions to Exponential Equations**

In a two-variable equation like , substitute values for x, and find the corresponding y to get the solution. For example, if , then .

**Math Facts**

When a number other than 0 is raised to the 0 power, it becomes 1, not 0.

**Solving for the Exponent in an Exponential Equation**

In a one-variable exponential equation where the exponent is unknown, isolate the exponential expression and then use guess and check.

**Example**

Solve for the equation .  
The last step was done by guess and check.

### Check Your Understanding of Section 8.1

1. Multiple-Choice
2. If and , solve for y.  
   **(2) 9**
3. If and , solve for y.  
   **(3) 54**
4. If and , solve for y.  
   **(3) 1**
5. If and , solve for y.  
   **(4) 5**
6. If , and , solve for y.  
   **(3)**
7. If and , solve for x.  
   **(4) 3**
8. If and , solve for x.  
   **(1) 7**
9. If and , solve for x.  
   **(2) -3**
10. If and , solve for y.  
    **(4) 8**
11. If and , solve for y rounded to the nearest hundredth.  
    **(1) 2.70**
12. Show how you arrived at your answers.
13. Phoebe put $500 into the bank. The amount of money she has after t years is determined by the equation . After 4 years, how much money will Phoebe have in the bank.  
      
    Using Scientific Calculator:
14. The population of a town after t years can be approximated by the equation   
    . (a) According to the formula, what will the population of the town be after 10 years? (b) In what year will the population become 14,065?  
      
    (a)   
      
    (b) 15th year
15. Zoe drinks a cup of coffee that has 100 mg of caffeine. The amount of caffeine in the bloodstream after t hours can be determined by the equation . How much caffeine will be left in her bloodstream after 20 hours?
16. Food that is 110 degrees is put into a 30-degree freezer. The temperature of the food is related to the number of hours the food is in the freezer by the equation   
    . Between which two hours will the food be 32 degrees?  
      
    **Between 10 and 11 hours.**
17. Daphne says that is always greater than . Julia says that this is not true and that sometimes is greater than . Which student is correct? Explain.  
      
    Julia is correct.  
      
    For , will be greater than .

x 5^x 6^x  
 -1 0.20000 0.16667

-2 0.04000 0.02778

-3 0.00800 0.00463

-4 0.00160 0.00077

-5 0.00032 0.00013

-6 0.00006 0.00002

## 8.2 Graphing Solution Sets to Two-Variable Exponential Equations

The solution set to a two-variable exponential equation, like , can be produced with a table or with a graphing calculator. The shape of the graph is not a line or a parabola but is a distinctive shape that looks a bit like a playground slide.

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A graph with a line drawn on it

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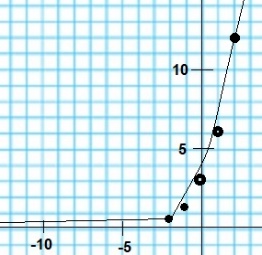
**Graphing the Solution Set to an Exponential Equation with a Table**

Five points are generally sufficient for graphing the solution set for an exponential equation. x-values of 2, 1, 0, -1, and -2 should produce enough points for an accurate graph.

**Example 1**

Make a table of solutions to , and use them to make a sketch of the graph of the solution set.

|  |  |
| --- | --- |
| x | y |
| 2 | 12 |
| 1 | 6 |
| 0 | 3 |
| -1 | 1.5 |
| -2 | 0.75 |



**Making Tables and Graphs on the Graphing Calculator**

**Example 2**

A blue rectangular object with black text

AI-generated content may be incorrect.

A graph with a line

AI-generated content may be incorrect.

**Math Facts**

In an exponential equation, the thing being raised to the power is called the *base*. When the base of an exponential equation is between 0 and 1, the graph shows exponential *decay*. When the base is greater than 1, the graph shows exponential *growth*.

### Check Your Understanding of Section 8.2

1. Multiple-Choice
2. Which ordered pair is in the solution set of ?  
   **(4) (4, 81)**
3. Which ordered pair is in the solution set of ?  
   **(3) (3, 40)**
4. What is the graph of   
   Points: (-1, 0.5), (0, 1), (2, 4), (3, 8)**(3)**
5. Which is the graph of ?  
   Points: (-1, 2), (0, 1), (1, 0.5), (2, 0.25)  
   **(1)**
6. Below is the graph of which equation?  
   Points (-1, 0.33), (0, 1), (1, 3), (2, 9)  
   **(4)**
7. Below is a graph of  
   Points: (-2, 9), (-1, 3), (0, 1), (1, 0.33)  
   **(1)**
8. The chart below has ordered pairs for which equation?

|  |  |
| --- | --- |
| x | y |
| 0 | 6 |
| 1 | 12 |
| 2 | 24 |
| 3 | 48 |

**(4)**

1. The chart below has ordered pairs for which equation?

|  |  |
| --- | --- |
| x | y |
| 0 | 6 |
| 1 | 3 |
| 2 | 1.5 |
| 3 | 0.75 |

**(2)**

1. In what interval is the graph of increasing?  
   **(1) Always**
2. Below is the graph of . What is true about the value of b?  
   **(2) b must be less than 1 and greater than 0.**
3. Show how you arrived at your answers.
4. The graph of passes through the point (5, 243). What must the value of b be?  
     
   **Answer: b = 3**

|  |  |
| --- | --- |
| x | y |
| 1 | 1 |
| 2 | 32 |
| 3 | 243 |

1. After putting $200 into the bank, the amount of money Xavier has after 5 years is . Make a graph showing how the money grows for 5 years.

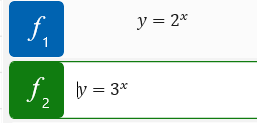
A graph with a line going up

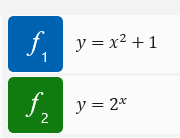
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1. Below is the graph of and on the same set of axes.  
     
   A graph of a function

   AI-generated content may be incorrect.  
     
   Is it true that for all values of x? Explain.  
     
   **No. for x less than 0.**

|  |  |  |
| --- | --- | --- |
| x | 2^x | 3^x |
| -1 | 0.50000 | 0.33333 |
| -2 | 0.25000 | 0.11111 |
| -3 | 0.12500 | 0.03704 |
| -4 | 0.06250 | 0.01235 |

1. What is the solution to this system of equations?  
   **Answer: x = 0, y = 1**  
     
   A graph of a function

   AI-generated content may be incorrect.
2. Below are the graphs of and   
   . They both contain the point (0, 1). Is it true that for all values of ? Explain.  
     
     
   A graph of a function

   AI-generated content may be incorrect.

The graphs intersect at points (0, 1) and (1, 2).  
From x = 0 to 1, .  
From x > 1, .  
  
Note: Book answer says x > 4.3, which appears to be incorrect.

## 8.3 Distinguishing Among Linear, Quadratic, and Exponential Equations

The graphs of the solution sets of linear, quadratic, and exponential equations are very distinctive. So it is possible to tell what type of equation a graph was A graph with a line and numbers in a chart

AI-generated content may be incorrect.produced from by just looking at it. When the solution set is in table form, it is possible to graph the data from the table or to determine which type of equation it came from with fast calculations.

**Identifying What Type of Equation It Is By Looking at the Graph of the Solution Set**

Linear equations have no exponents great than or equal to 1. The graph of a linear equation is always a line.

Quadratic equations have either an -term or a -term. The graph of a quadratic equation is a parabola.

Exponential equations have an x as an exponent. The graph of a exponential equation looks like a playground slide.

**Identifying That an Equation Is Exponential By Looking at a Table of Values**

**Using the Graphing Calculator to Graph Tables of Values**

**Example 4**

**Identifying the Type of Equation from a Table Without Graphing**

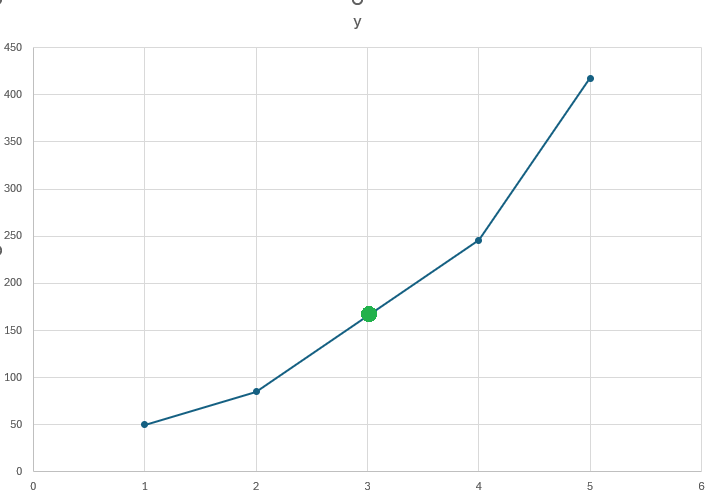
Since all the differences are the same, this is a linear equation.

A graph with a line and numbers in a chart

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### Check Your Understanding of Section 8.3

1. Multiple-Choice
2. What type of equation has a graph like the one below?  
   **(1) Linear**
3. Why type of equation has a graph like the one below?  
   **(1) Quadratic**
4. What type of equation has a graph like the one below?  
   **(2) Exponential**
5. Which of the graphs below corresponds to a quadratic equation?  
   **(3)**
6. Which of the graphs below corresponds to an exponential equation?  
   **(2)**
7. The ordered pairs in the following chart satisfy which type of equation?  
   **(1) Linear**
8. Which type of graph below corresponds to a linear equation?  
   **(1)**
9. The ordered pairs in the following chart satisfy which type of equation?  
   **(3) Exponential**
10. The ordered pairs in the following chart satisfy which type of equation?  
    **(4) None of the above**
11. The following scatterplot can be best modeled with which type of equation?  
    **(3) Quadratic**
12. Show how you arrived at your answers.
13. The population of a county for several different times is plotted on a graph where the x-axis represents years since the year 1990 and the y-axis represents the population in millions. The graph is below. What type of equation could be used to model this data? Explain.  
      
    An exponential equation can be best used to explain population increases, because population tends to increase by the same percentage each year.  
    The graph is increasing each year.
14. Some data points on a graph are plotted. Myah believes that this is a portion of a quadratic graph. Chloe believes that this is a portion of an exponential graph. Is there enough information to determine who is correct? If not, what more information would be needed?  
      
    Both Myah and Chloe could be correct, as the points could reflect that of a quadratic or exponential graph. There is not enough information.  
      
    More data about y-values for negative x-values would be helpful in determining whether it is a quadratic or exponential graph.
15. When the number of seats o na train is graphed as the y-coordinate and the time since 8:00 am is graphed as the x-coordinate, it makes the graph below. Which type of equation could be used to model this data? What details do you know about the equation besides just the type of equation it is?  
      
    It appears to be an exponential decay graph, because it is a curve that starts at a high value and decreases rapidly, approaching but never reaching the x-axis (horizontal asymptote), showing a decreasing trend over time.  
      
    For an exponential decay graph, the base will be between 0 and 1.  
      
    The starting value at time zero appears to be 100. It approaches nearly zero at 20 minutes.
16. Below is a chart with ordered pairs that satisfy an equation. The equation is linear, quadratic or exponential. Determine which type of equation it is, and fill in the missing number.  
      
    A screenshot of a calculator

    AI-generated content may be incorrect.  
      
    The type of equation is exponential.  
      
    The missing point is estimated to be (3, 150) to (3, 165).  
      
    
17. A tennis ball is dropped from a height of 30 feet. After each bounce the highest point is 80% as heigh as the bounce before. If the height of each bounce is graphed as the y-coordinate and the bounce number is the x-coordinate, will the graph be linear, quadratic or exponential.  
      
    The graph will be an exponential decay graph with the initial point of (0, 30).  
      
    A graph with a blue line

    AI-generated content may be incorrect.

## 8.4 Real-World Problems Involving Exponential Equations

Exponential equations can be used to model many real-life situations ranging from the way that a cup of hot chocolate cools down to the way that the world population increases. If you are given an exponential equation that is a model for something in the real world, that equation can be used to answer questions about the real world.

**Modeling a Population Increase**

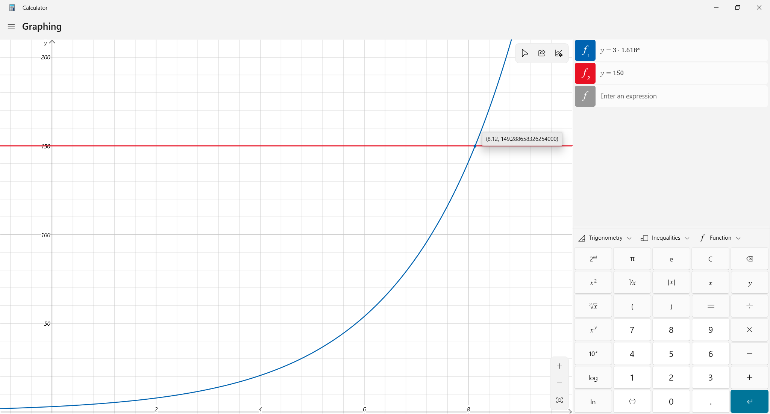
In a certain forest, the population of rabbits, in thousands, x years from now can be modeled by the equation . With this equation, there are two main questions that can be asked: (1) What will the population be after a certain number of years? (2) In what year will the rabbit population reach a certain number?

What will the poplation be in 5 years?

How many years will it take for the rabbit population to reach 150 thousand rabbits? After year 8.

|  |  |
| --- | --- |
| **Year** | **Population** |
| 5 | 33.26702 |
| 6 | 53.82603 |
| 7 | 87.09052 |
| 8 | 140.91246 |
| 9 | 227.99636 |
| 10 | 368.89811 |

The rabbit population will reach 150 thousand rabbits after 8.12 years.



**Example**

The temperature of a slice of pizza that just came out of the oven can be modeled by the equation   
, where y is the temperature in degrees and x is the number of minutes since the pizza was removed from the oven. What will the temperature be after 10 minutes?

### Check Your Understanding of Section 8.4

1. Multiple-Choice
2. The population of a country can be modeled with the equation , where P is the population in millions and t is the number of years since 2010. According to this model, rounded to the nearest 10 million, what will the population of this country be in 2019.  
   **(2) 460,000,000**
3. The amount of carbon-14 remaining in a substance can be calculated with the formula , where t is the time in years, P is the original amount of carbon-14 remaining. If the bone in a living animal has 200 grams of carbon-14, how many grams of carbon-14 will remain 20,000 years after it dies rounded to the nearest tenth of a gram?  
   **(3) 17.8 grams**
4. Kyle puts $300 into a savings account. The amount of money in the account after t years can be calculated with the formula   
   , where P is the original amount of money and A is the amount of money after t years. How much money will Kyle have after 10 years?  
   **(4) $403.17**
5. A ball is dropped from a window 30 feet above the ground. The height of the nth bounce can be calculated with the formula   
   , where h is the height at the top of the nth bounce. How high will the ball go after the sixth bounce?  
   **(1) 3.5 feet**
6. A cake put into a 400-degree oven from an 80 degree kitchen heats up according to the equation where t is the temperature of the food and m is the number of minutes since it was put into the oven. How hot will the food be after 5 minutes in the oven?  
   **(2) 295 degrees**
7. A cup of tea that is 200 degrees is put into a room that is 80 degrees. The temperature of the tea can be calculated with the formula where m is the number of minutes since the tea was put into the room. What will the temperature of the tea be after 10 minutes?  
   **(4) 122 degrees**
8. The grade jerry gets on a test is related to the number of hours he studies by the equation  
    where h is the number of hours studied and g is the grade the student gts. According to this equation, what grade will Jerry get if he studies for 6 hours?  
   **(3) 93**
9. Jude weighs 250 pounds and goes on a diet. His weight is etermined by the equation  
    where t is the number of weeks since he started the diet and p is his weight in pounds. How much will Jude weigh after 12 weeks?  
   **(3) 213 pounds**
10. A YouTube video of a cat wearing rain boots goes viral. The number of views d days after the video was posted can be calculated with the equation . How many times will the video be viewed 20 days after it was posted, rounded to the nearest million?  
    **(1) 2,000,000**
11. The number of shaded rectangles in the first picture is 3. The number of shaded rectangles in the second picture is 9. If this pattern continues, the number of shaded triangles can be calculated with the formula . How many shaded rectangles would be in the fifth picture?  
    **(3) 243**
12. *Show how you arrived at your answers*.
13. The population of a country can be determined by the equation  
     where P is the population in millions and t is the number of years since 2010. (a) Use this formula to determine the population of the country in 2019. (b) Use this formula to determine when the population of the country will be 800 million.  
    (a) = **901 million**  
    (b)  **million (shortly after year 8 – 2018)**
14. Ice cream is removed from a freezer and put into an 80 degree room. The formula that can be used to determine the temperature of the ice cream m minutes after being removed from the freezer is . (a) Use this formula to determin what the temperature of the ice cream will be after 5 minutes. (b) After how many minutes will the ice cream be 72 degrees?  
      
    (a) (b) **17 minutes**

|  |  |
| --- | --- |
| **Minutes** | **Temperature** |
| 10 | 63.26343 |
| 11 | 64.93709 |
| 12 | 66.44338 |
| 13 | 67.79904 |
| 14 | 69.01914 |
| 15 | 70.11723 |
| 16 | 71.10550 |
| 17 | 71.99495 |
| 18 | 72.79546 |
| 19 | 73.51591 |
| 20 | 74.16432 |

1. Lucas wants to put $100 into a bank. For the first bank, the formula for how much money he will have after t years is where P is the amount of money originally deposited. For the second bank, the formula for how much money he will have after t years is . (a) If he is going to withdraw the money after 10 years, which bank is the better choice? (b) If he is going to withdraw his money after 54 years, which bank is the better choice?  
   (a) (b)
2. The first picture has 4 line segments. The second picture has 16 line segments. If the equation for the number of line segments of the nth picture is , how many line segments will be in the fifth picture?
3. Zion has a goal to bench press 250 pounds. The equation that can be used to determine the amount he can bench press after t weeks of exercise is . (a) How much will he be able to bench press after 5 weeks? (b) When will he be able to bench press 250 pounds?  
   (a)  **pounds  
   (b) Week 21**

|  |  |
| --- | --- |
| **Weeks** | **Bench Press** |
| 15 | 225.9 |
| 16 | 231.1 |
| 17 | 235.9 |
| 18 | 240.4 |
| 19 | 244.6 |
| 20 | 248.5 |
| 21 | 252.1 |